Trigomania

More than you ever wanted to know about trigonometry

Rectangular Coordinate System

\( (x, y) \)

\( x \)

\( y \)

Rectangular Coordinate System

\( (3, 5) \)

\( x \)

\( y \)
Rectangular Coordinate System

Quadrant I
Quadrant II
Quadrant III
Quadrant IV

Pythagorean Theorem

\[ a^2 + b^2 = c^2 \]

Pythagorean Theorem

\[ a = \sqrt{c^2 - b^2} = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7} = 3.6 \]
### Basic Trig Functions

- **Sin** $\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- **Cos** $\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- **Tan** $\theta = \frac{\text{opposite}}{\text{adjacent}}$

**Soh-Cah-Toa**

### Example 1

- $\sin 30^\circ = \frac{2}{4} = 0.5$
- $\theta = \sin^{-1}(0.5) = 30^\circ$

### Example 2

- $b = 5(\cos 35^\circ) = 5(0.819) = 4$

**Adjacent side**

**Hypotenuse**

**Opposite side**
Trig Relationships

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]
\[ \tan \theta = \frac{\text{opp/hyp}}{\text{adj/hyp}} \]
\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]

Complement Angles

\[ 90^\circ + \theta + \phi = 180^\circ \]
\[ \theta + \phi = 90^\circ \]
\[ \phi = 90^\circ - \theta \]
\[ \phi = 90^\circ - \theta \]
Complement Angles

opposite for $\theta$ is adjacent for $\varphi$ and vice versa

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \cos \varphi
\]

\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \sin \varphi
\]

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \cot \varphi
\]

Polar Coordinate System

\[
\sin \theta = \frac{y}{r} \Rightarrow y = r \sin \theta
\]

\[
\cos \theta = \frac{x}{r} \Rightarrow x = r \cos \theta
\]

Polar Coordinate System

\[
\sin \theta = \frac{y}{r} \Rightarrow \sin \theta \text{ is positive}
\]

\[
\cos \theta = \frac{-x}{r} \Rightarrow \cos \theta \text{ is negative}
\]
Polar Coordinate System

<table>
<thead>
<tr>
<th>Sign of</th>
<th>Quad I (0 ≤ θ ≤ 90)</th>
<th>Quad II (90 ≤ θ ≤ 180)</th>
<th>Quad III (180 ≤ θ ≤ 270)</th>
<th>Quad IV (270 ≤ θ ≤ 360)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>y</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>sin θ</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>cos θ</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>tan θ</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Supplement Angle

Quad II

\[-x, y\]

\[\theta\]

\[r\]

\[x\]

\[y\]

\[\theta + a = 180^\circ \Rightarrow a = 180^\circ - \theta\]

\[\sin \theta = y/r = \sin a\]

\[\cos \theta = -x/r = -\cos a\]

\[\tan \theta = y/-x = -\tan a\]

Supplement Angle

Quad III

\[-x, -y\]

\[\theta\]

\[r\]

\[x\]

\[y\]

\[\theta = 180^\circ + a \Rightarrow a = 180^\circ - \theta\]

\[\sin \theta = y/r = \sin a\]

\[\cos \theta = -x/r = -\cos a\]

\[\tan \theta = y/-x = \tan a\]
Supplement Angle

\[
\begin{align*}
\theta &= 360^\circ - \alpha \\
\sin \theta &= \frac{y}{r} = -\sin \alpha \\
\cos \theta &= \frac{x}{r} = \cos \alpha \\
\tan \theta &= \frac{-y}{x} = -\tan \alpha \\
\end{align*}
\]

What is it good for?

- Extends the trig functions to any angle — positive or negative and as large as you want it.
- Allows you to quickly check the sign of the trig functions for any angle by noting what quadrant it is in.
- Enables you to visualize things like: \( \cos 120^\circ = -\cos 60^\circ = -\sin 30^\circ \) which allows you to rewrite expressions in forms that are easier to work with.

Special Triangles
Special Triangles

<table>
<thead>
<tr>
<th>θ</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>0.5</td>
<td>(\sqrt{3}/2)</td>
<td>1(\sqrt{3})=0.577</td>
</tr>
<tr>
<td>45°</td>
<td>1/(\sqrt{2})=0.707</td>
<td>1/(\sqrt{2})=0.707</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>(\sqrt{3}/2) = 0.866</td>
<td>0.5</td>
<td>(\sqrt{3}/1)=1.732</td>
</tr>
<tr>
<td>90°</td>
<td>1</td>
<td>0</td>
<td>infinite</td>
</tr>
</tbody>
</table>