

27 The Electric Field

27.1 Electric Field Models

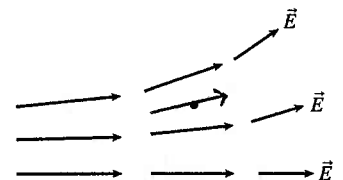
27.2 The Electric Field of Multiple Point Charges

1. You've been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.

A tiny, positive test charge will be placed at the point in space and the force will be measured. From the force measurement and the charge, the electric field will be calculated using $E = \frac{F}{q}$. The direction of the field will be the same as the direction of the force because q is positive.

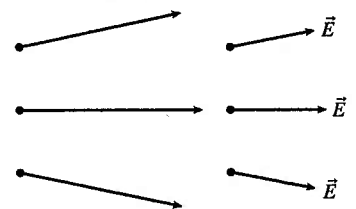
2. Is there an electric field at the position of the dot? If so, draw the electric field vector on the figure. If not, what would you need to do to create an electric field at this point?

Yes there is an electric field at this position.



3. This is the electric field in a region of space.
 - a. Explain the information that is portrayed in this diagram.

There is some source of positive charge on the left side. The longer vectors indicate a relatively larger electric field on the left, closest to the charge.

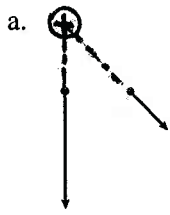


- b. If field vectors were drawn at the same six points but each was only half as long, would the picture represent the same electric field or a different electric field? Explain.

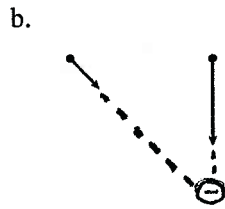
It could represent the same electric field because the length of the field vector only represents relative magnitude of the electric field at that point.

4. Each figure shows two vectors. Can a point charge create an electric field that looks like this at these two points? If so, add the charge to the figure. If not, why not?

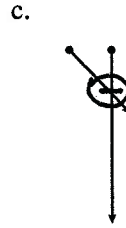
Note: The dots are the points to which the vectors are attached. There are no charges at these points.



Yes. Point 2 is further away from charge.



Yes.

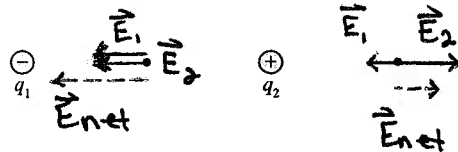
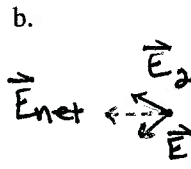
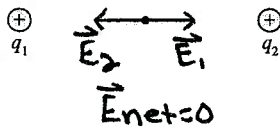
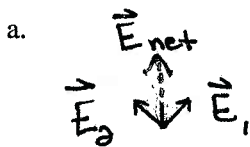


Yes. The field vector shows the field only at the dot.

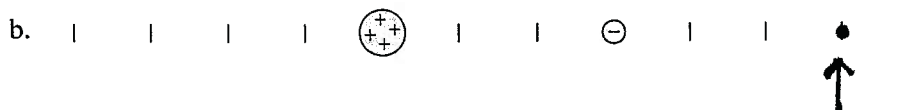
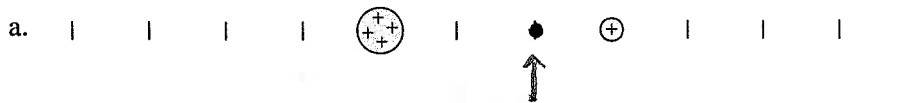


No. The field due to a point charge cannot go in two directions.

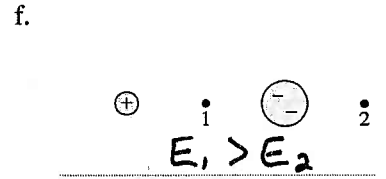
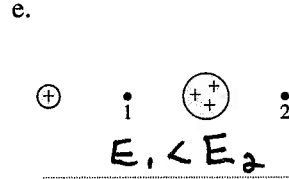
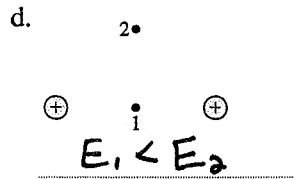
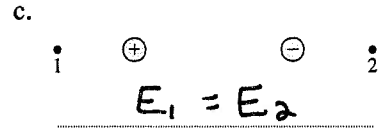
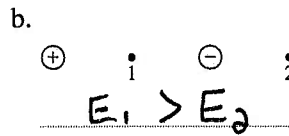
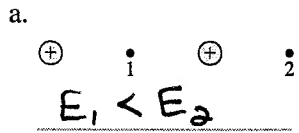
5. At each of the dots, use a **black** pen or pencil to draw and label the electric fields \vec{E}_1 and \vec{E}_2 due to the two point charges. Make sure that the *relative* lengths of your vectors indicate the strength of each electric field. Then use a **red** pen or pencil to draw and label the net electric field \vec{E}_{net} .



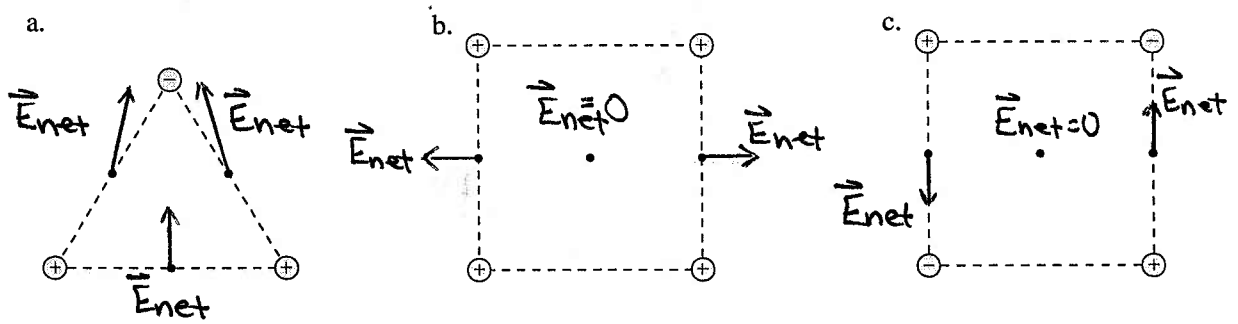
6. For each of the figures, use dots to mark any point or points (other than infinity) where $\vec{E} = \vec{0}$.



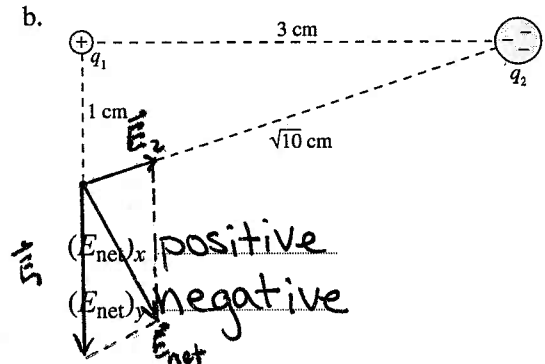
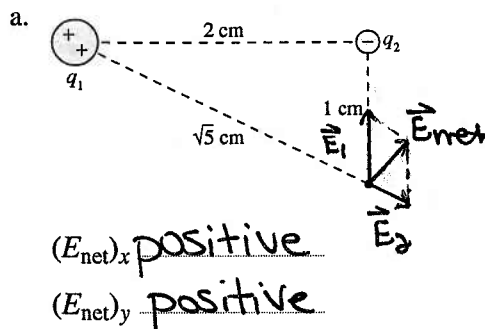
7. Compare the electric field strengths E_1 and E_2 at the two points labeled 1 and 2. For each, is $E_1 > E_2$, is $E_1 = E_2$, or is $E_1 < E_2$?



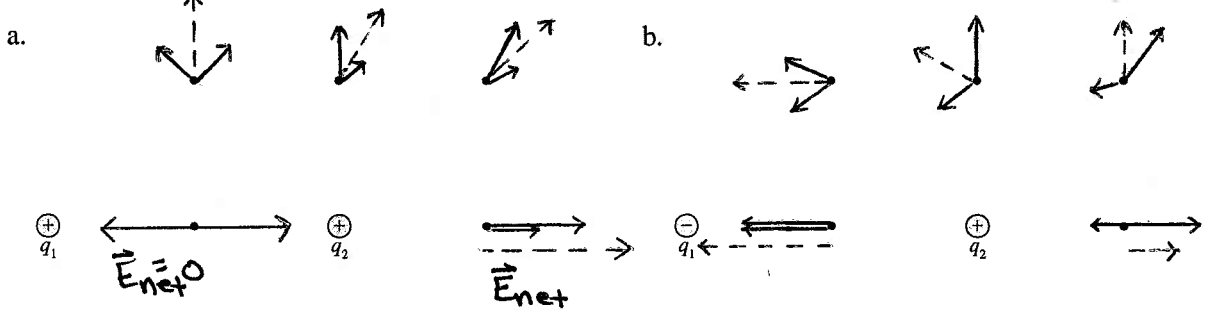
8. For each figure, draw and label the net electric field vector \vec{E}_{net} at each of the points marked with a dot or, if appropriate, label the dot $\vec{E}_{\text{net}} = \vec{0}$. The lengths of your vectors should indicate the magnitude of \vec{E} at each point.



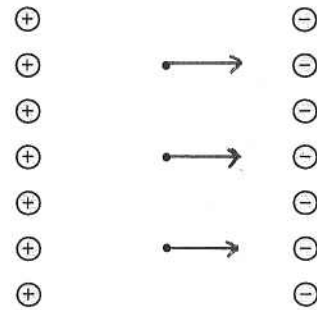
9. At the position of the dot, draw field vectors \vec{E}_1 and \vec{E}_2 due to q_1 and q_2 , and the net electric field \vec{E}_{net} . Then, in the blanks, state whether the x - and y -components of \vec{E}_{net} are positive or negative.



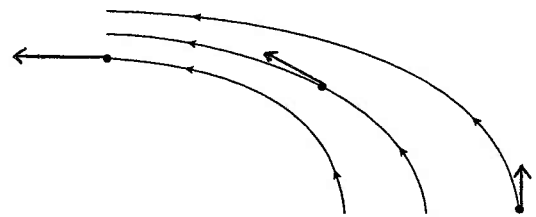
10. Use a **black pen or pencil** to draw the two electric fields \vec{E}_1 and \vec{E}_2 at each dot. Then use a **red pen or pencil** to draw \vec{E}_{net} . The lengths of your vectors should indicate the magnitude of \vec{E} at each point.



11. Draw the electric field vector at the three points marked with a dot.
Hint: Think of the charges as horizontal positive/negative pairs, then use superposition.



12. The figure shows the electric field lines in a region of space. Draw the electric field vectors at the three dots.

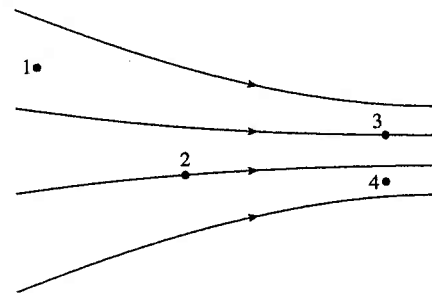


13. The figure shows the electric field lines in a region of space. Rank in order, from largest to smallest, the electric field strengths E_1 to E_4 at points 1 to 4.

Order: $E_3 = E_4 > E_2 > E_1$

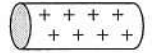
Explanation:

The electric field strength is larger in the region where the field lines are closer together (E_3 and E_4) and smaller where the field lines are farther apart.



27.3 The Electric Field of a Continuous Charge Distribution

14. A small segment of wire contains 10 nC of charge.



a. The segment is shrunk to one-third of its original length. What is the ratio λ_f/λ_i , where λ_i and λ_f are the initial and final linear charge densities?

$$\frac{\lambda_f}{\lambda_i} = \frac{(Q_f/L_f)}{(Q_i/L_i)} \text{ But } Q_i = Q_f \Rightarrow \frac{\lambda_f}{\lambda_i} = \frac{L_i}{L_f} = 3$$

b. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be added to the wire to keep the linear charge density unchanged?

10 times the original amount of charge would give a constant linear charge density. So the amount of charge to add to the original is 9 times the original charge.

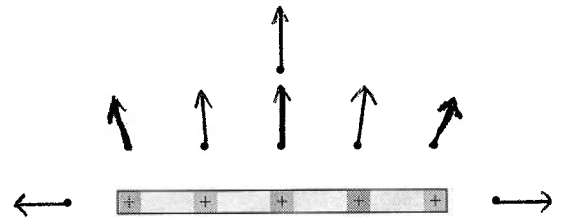
15. A wire has initial linear charge density λ_i . The wire is stretched in length by 50%, and one-third of the charge is removed. What is the ratio λ_f/λ_i , where λ_f is the final linear charge density?

$$\lambda_i = \frac{Q_i}{L_i} \quad L_f = 1.5 L_i \quad Q_f = \frac{Q_i}{3}$$

$$\lambda_f = \frac{Q_f}{L_f} = \frac{(Q_i/3)}{(1.5 L_i)} = \frac{1}{(3)(1.5)} \frac{Q_i}{L_i} = \frac{1}{4.5} \lambda_i$$

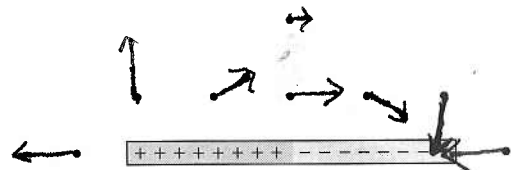
$$\frac{\lambda_f}{\lambda_i} = \frac{1}{4.5}$$

16. The figure shows a uniformly charged positive wire. Five small, equally-spaced segments of charge are shown. Use these five segments to estimate the wire's electric field—both magnitude and direction—at each point in space marked with a dot. Draw each \vec{E} on the figure.

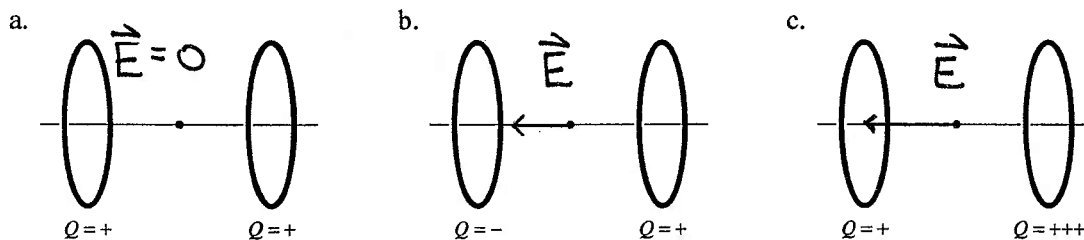


17. Equal-length, equally charged positive and negative wires are placed end-to-end. Draw the electric field at each of the dots.

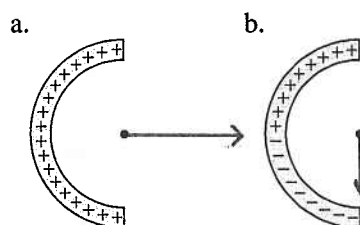
Hint: Think about the superposition of the fields of a positive and a negative wire.



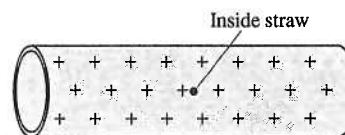
18. Two rings of charge face each other. The total charge on each ring is indicated beneath it. Draw the electric field vector on the axis of the rings at the midpoint between them (at the dot), or label the point $\vec{E} = \vec{0}$.



19. The figure shows two charged rods bent into a semicircle. For each, draw the electric field vector at the “center” of the semicircle.



20. A hollow soda straw is uniformly charged. What is the electric field at the center (inside) of the straw? Explain.



The electric field at the center is zero. We can think of the straw as being made up of as many rings of positive charge. At the center of the ring adding all field vectors gives a resultant electric field equal to zero.



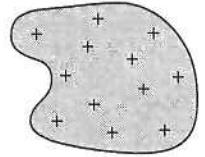
21. An electron experiences a force of magnitude F when it is 1 cm from a very long charged wire with linear charge density λ . If the charge density is doubled, at what distance from the wire will a proton experience a force of the same magnitude F ?

$$F = eE = e \left(\frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \right)$$

If the charge density λ is doubled, then the distance r from the wire must also be doubled for the force to be the same. $r = 2\text{ cm}$

27.4 The Electric Fields of Rings, Disks, Planes, and Spheres

22. An irregularly-shaped area of charge has surface charge density η_i . Each dimension (x and y) of the area is reduced by a factor of 3.163.



- a. What is the ratio η_f/η_i , where η_f is the final surface charge density?

$$A_f = \frac{A_i}{3.163^2} \quad \frac{\eta_f}{\eta_i} = \frac{(Q/A_f)}{(Q/A_i)} = \frac{A_i}{A_f} = 3.163^2 = 10.00$$

- b. Compare the final force on an electron very far away to the initial force on the same electron.

$$F_f = eE_f = e \frac{\eta_f}{\epsilon_0} \quad F_i = eE_i = e \frac{\eta_i}{\epsilon_0} \quad \frac{F_f}{F_i} = \frac{\eta_f}{\eta_i} = 9.99$$

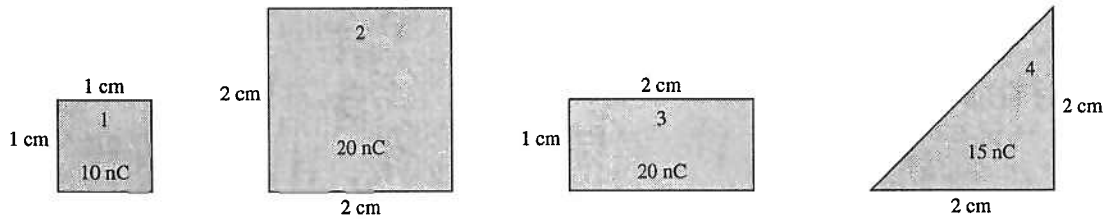
$$F_f = 9.99 F_i$$

23. A circular disk has surface charge density 8 nC/cm^2 . What will be the surface charge density if the radius of the disk is doubled?

$$\eta_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2} = 8 \frac{\text{nC}}{\text{cm}^2}$$

$$\eta_2 = \frac{Q}{A_2} = \frac{Q}{\pi (2r_1)^2} = \frac{Q}{4\pi r_1^2} = \frac{1}{4} \eta_1 = 2 \frac{\text{nC}}{\text{cm}^2}$$

24. Rank in order, from largest to smallest, the surface charge densities η_1 to η_4 of surfaces 1 to 4.



Order: $\eta_1 = \eta_3 > \eta_4 > \eta_2$

Explanation:

$$\eta_1 = \frac{Q_1}{A_1} = \frac{10 \text{ nC}}{(1 \text{ cm} \times 1 \text{ cm})} = 10 \frac{\text{nC}}{\text{cm}^2} \quad \eta_2 = \frac{20 \text{ nC}}{(2 \text{ cm} \times 2 \text{ cm})} = 5 \frac{\text{nC}}{\text{cm}^2}$$

$$\eta_3 = \frac{20 \text{ nC}}{(1 \text{ cm} \times 2 \text{ cm})} = 10 \frac{\text{nC}}{\text{cm}^2} \quad \eta_4 = \frac{15 \text{ nC}}{\frac{1}{2}(2 \text{ cm} \times 2 \text{ cm})} = 7.5 \frac{\text{nC}}{\text{cm}^2}$$

25. A sphere of radius R_i has charge Q_i . What happens to the electric field strength at $r = 2R_i$ if:

- a. The quantity of charge is halved?

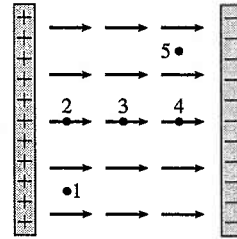
$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{If } Q \text{ is halved, then } E \text{ is also halved.}$$

- b. The radius of the sphere is halved?

The field outside a sphere is the same as that of a point charge Q located at the center of the sphere. So if the radius of the sphere changes, the field remains the same outside the sphere at the distance $r = 2R_i$.

27.5 The Parallel-Plate Capacitor

26. Rank in order, from largest to smallest, the electric field strengths E_1 to E_5 at each of these points.



Order: $E_1 = E_2 = E_3 = E_4 = E_5$

Explanation:

The electric field is constant everywhere between the plates. This is indicated by the electric field vectors which are all the same length and in the same direction.

27. A parallel-plate capacitor is constructed of two square plates, size $L \times L$, separated by distance d . The plates are given charge $\pm Q$. What is the ratio E_f/E_i of the final electric field strength E_f to the initial electric field strength E_i if:

a. Q is doubled?

$$\frac{E_f}{E_i} = \frac{\eta_f/\epsilon_0}{\eta_i/\epsilon_0} = \frac{\eta_f}{\eta_i} = \frac{Q_f/A_f}{Q_i/A_i} \quad \text{If } Q \text{ is doubled (} A = \text{constant)}$$

$$\frac{E_f}{E_i} = \frac{Q_f}{Q_i} = 2$$

b. L is doubled?

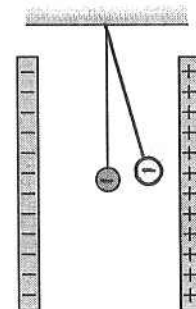
$$\text{If } L \text{ is doubled then } A_f = 4A_i \text{ (} Q = \text{constant)}$$

$$\frac{E_f}{E_i} = \frac{A_i}{A_f} = \frac{A_i}{4A_i} = \frac{1}{4}$$

c. d is doubled?

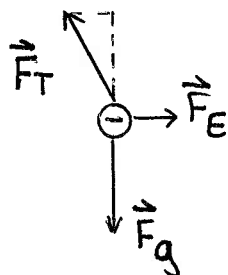
$$E \text{ does not depend on } d. \quad \frac{E_f}{E_i} = 1$$

28. A ball hangs from a thread between two vertical capacitor plates. Initially, the ball hangs straight down. The capacitor plates are charged as shown, then the ball is given a small negative charge. The ball moves to one side, but not enough to touch a capacitor plate.



a. Draw the ball and thread in the ball's new equilibrium position.

b. In the space below, draw a free-body diagram of the ball when in its new position.

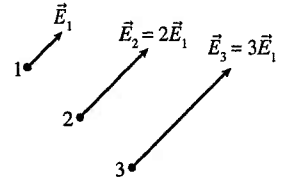


27.6 Motion of a Charged Particle in an Electric Field

27.7 Motion of a Dipole in an Electric Field

29. A small positive charge q experiences a force of magnitude F_1 when placed at point 1. In terms of F_1 :

- a. What is the force on charge q at point 3? 3F₁
- b. What is the force on a charge $3q$ at point 1? 3F₁
- c. What is the force on a charge $2q$ at point 2? 4F₁
- d. What is the force on a charge $-2q$ at point 2? -4F₁



30. A small object is released from rest in the center of the capacitor. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?

a. Positive object.

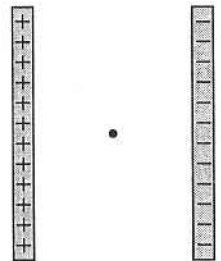
Accelerates to the right.

b. Negative object.

Accelerates to the left.

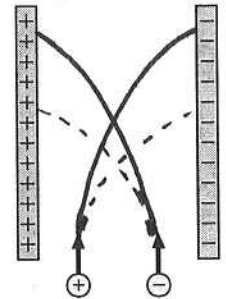
c. Neutral object.

Remains in place.

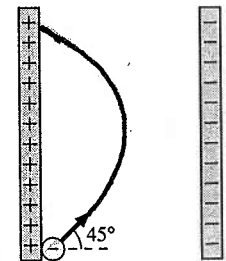


31. Positively and negatively charged objects, with equal masses and equal quantities of charge, enter the capacitor in the directions shown.

- a. Use solid lines to draw their trajectories on the figure if their initial velocities are fast.
- b. Use dashed lines to draw their trajectories on the figure if their initial velocities are slow.



32. An electron is launched from the positive plate at a 45° angle. It does not have sufficient speed to make it to the negative plate. Draw its trajectory on the figure.



33. A proton and an electron are released from rest in the center of a capacitor.
 a. Compare the forces on the two charges. Are they equal, or is one larger? Explain.

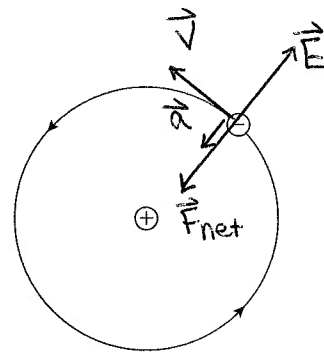
The forces on the two charges are equal.
 $F = qE$ They each have the same amount of charge and are placed in the same field.

- b. Compare the accelerations of the two charges. Are they equal, or is one larger? Explain.

The acceleration of the electron is larger because the electron has smaller mass. $a = \frac{F}{m}$

34. The figure shows an electron orbiting a proton in a hydrogen atom.
 a. What force or forces act on the electron?

The electric force.



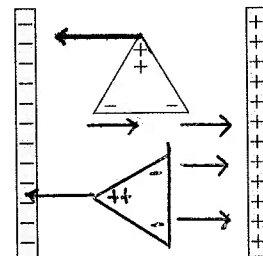
- b. Draw and label the following vectors on the figure: the electron's velocity \vec{v} and acceleration \vec{a} , the net force \vec{F}_{net} on the electron, and the electric field \vec{E} at the position of the electron.

35. Does a charged particle always move in the direction of the electric field? If so, explain why. If not, give an example that is otherwise.

No. If the charged particle has an initial velocity component in a perpendicular direction then it would travel in a different direction than the field. For example, a charge could move in circular motion as in problem # 34 above.

36. Three charges are placed at the corners of a triangle. The ++ charge has twice the quantity of charge of the two - charges; the net charge is zero.

- a. Draw the force vectors on each of the charges.
 b. Is the triangle in equilibrium? **No** If not, draw the equilibrium orientation directly beneath the triangle that is shown.
 c. Once in the equilibrium orientation, will the triangle move to the right, move to the left, rotate steady, or be at rest? Explain.



In equilibrium the triangle will remain in place because the net force is zero and the net torque is zero.