Physics 102 RC Circuit Lab – Answers to Related Exercises

1) The figure below shows a simple RC circuit consisting of a 100.0-V battery in series with a 10.0-µF capacitor and a resistor. Initially, the switch S is open and the capacitor is uncharged. Two seconds after the switch is closed, the voltage across the capacitor is 37 V.

a. What is the numerical value of the resistance R?
b. How much charge is on the capacitor 2.0 s after the switch is closed?

Given: \( V_0 = 100 \text{ V}, \quad C = 10.0 \text{ µF} = 10 \times 10^{-6} \text{ F}, \quad t = 2 \text{ s}, \quad V = 37 \text{ V} \)

\( \text{a) The capacitor is charging, so the equation to use is:} \quad V = V_0 [1 - e^{(-t/RC)}] \text{ and solve for } R: \)

\[
\frac{V}{V_0} = 1 - e^{(-t/RC)}
\]

\[
e^{(-t/RC)} = 1 - \frac{V}{V_0}
\]

\[
(-t/RC) = \ln \left( 1 - \frac{V}{V_0} \right)
\]

\[
R = \frac{-t}{C \ln \left( 1 - \frac{V}{V_0} \right)} = \frac{-2s}{(10 \times 10^{-6} \text{ F}) \ln (1 - 37V/100V)} = 433,000 \Omega
\]

\( \text{b) The charge on any capacitor is always equal to its capacitance times the voltage across it: } q = CV \)

\[
q = (10 \times 10^{-6} \text{ F})(37 \text{ V}) = 3.7 \times 10^{-4} \text{ C}
\]

2) An uncharged 5.0-µF capacitor and a resistor are connected in series to a 12-V battery and an open switch to form a simple RC circuit. The switch is closed at \( t = 0 \text{ s} \). The time constant of the circuit is 4.0 s.

a. Determine the value of the resistance R.
b. Determine the maximum charge to which the capacitor can be charged.
c. What is the charge on either plate after one time constant has elapsed?

Given: \( V_0 = 12 \text{ V}, \quad C = 5 \text{ µF} = 5 \times 10^{-6} \text{ F}, \quad \tau = 4 \text{ s} \)

\( \text{a) } \tau = RC \Rightarrow R = \tau/C = 4s/(5 \times 10^{-6} \text{ F}) = 8 \times 10^5 \Omega \)

\( \text{b) max } q = CV_0 \) (see reasoning in 1b) \( \Rightarrow q = (5 \times 10^{-6} \text{ F})(12V) = 6 \times 10^{-5} \text{ C} \)

\( \text{c) Use the charging equation again. (Same as in 1a) } \Rightarrow V = V_0 [1 - e^{(-t/RC)}] \)

\( \text{But if } t = \tau, \text{ then } t = RC \) (since \( \tau = RC \)), so:

\[
V = V_0 [1 - e^{(-RC/RC)}] = V_0 [1 - e^{(-1)}] = V_0 [0.632]
\]

\[
V = 12V(0.632) = 7.59 \text{ V} \text{ and } q = CV, \text{ so } q = (5 \times 10^{-6} \text{ F})(7.59 \text{ V}) = 3.8 \times 10^{-5} \text{ C}
\]
3) An RC circuit as shown below, consists of a resistor with resistance 1.0 kΩ, a 120-V battery, and two capacitors, C1 and C2, with capacitances of 20.0 µF and 60.0 µF, respectively. Initially, the capacitors are uncharged; and the switch is closed at t = 0 s.

a. What is the current through the resistor a long time after the switch is closed? Recall that current is the charge per unit time that flows in a circuit.

b. What is the time constant of the circuit?

c. Determine the total charge on both capacitors two time constants after the switch is closed.

\[ a) I = \frac{\Delta q}{\Delta t}, \text{when the capacitor is fully charged, } \Delta q = 0, \text{ so } I = 0 \]

(when a capacitor is fully charged, it acts like a break in the circuit unless you short circuit the battery, then charge flows in the opposite direction of the original current)

\[ b) \text{Given: } R = 1 \text{ kΩ}, V_o = 120 \text{ V}, C_{eq} = 20 \mu F + 60 \mu F = 80 \mu F \]

\[ \tau = RC = (1 \text{ kΩ})(80 \mu F) = 0.08 \text{ s} \]

\[ c) q = CV \text{ for any capacitor. To find } V, \text{ we'll use the charging equation: } V = V_o [1 - e^{(-t/RC)}] \]

Since \( \tau = RC \) we can write it as: \( V = V_o [1 - e^{(-t/\tau)}] \)

And when \( t = 2\tau \), the equation becomes: \( V = V_o [1 - e^{(-2)}] \)

\( V = 120 \text{ V} (0.865) = 103.76 \text{ V} \)

\[ q = CV = (80 \times 10^{-6} \text{ F})(103.76 \text{ V}) = 8.3 \times 10^{-3} \text{ C} \]