Physics 102 Team Session for Chapter 22 &23 Key

1) A generator works by rotating a coil of $N$ turns in a magnetic field $B$. It has a peak emf of 75V when the coil rotates at a frequency of 280 Hz.

a) What is the peak emf of the generator when the frequency is 45 Hz?

b) What is the frequency of the generator when the peak emf is 180 V?

**REASONING** The peak emf $\xi_0$ of a generator is found from $\xi_0 = NAB\omega$ (Equation 22.4), where $N$ is the number of turns in the generator coil, $A$ is the coil’s cross-sectional area, $B$ is the magnitude of the uniform magnetic field in the generator, and $\omega$ is the angular frequency of rotation of the coil. In terms of the frequency $f$ (in Hz), the angular frequency is given by $\omega = 2\pi f$ (Equation 10.6). Substituting Equation 10.6 into Equation 22.4, we obtain

$$\xi_0 = NAB(2\pi f) = 2\pi NABf \quad (1)$$

When the rotational frequency $f$ of the coil changes, the peak emf $\xi_0$ also changes. The quantities $N$, $A$, and $B$ remain constant, however, because they depend on how the generator is constructed, not on how rapidly the coil rotates. We know the peak emf of the generator at one frequency, so we will use Equation (1) to determine the peak emf for a different frequency in part (a), and the frequency needed for a different peak emf in part (b).

**SOLUTION**

a. Solving Equation (1) for the quantities that do not change with frequency, we find that

$$\frac{\xi_0}{f} = \frac{2\pi NAB}{\text{Same for all frequencies}} \quad (2)$$

The peak emf is $\xi_{0,1} = 75$ V when the frequency is $f_1 = 280$ Hz. We wish to find the peak emf $\xi_{0,2}$ when the frequency is $f_2 = 45$ Hz. From Equation (2), we have that

$$\frac{\xi_{0,2}}{f_2} = \frac{2\pi NAB}{\text{Same for all frequencies}} = \frac{\xi_{0,1}}{f_1} \quad (3)$$

Solving Equation (3) for $\xi_{0,2}$, we obtain

$$\xi_{0,2} = \left(\frac{f_2}{f_1}\right)\xi_{0,1} = \left(\frac{45 \text{ Hz}}{280 \text{ Hz}}\right)(75 \text{ V}) = 12 \text{ V}$$

b. Letting $\xi_{0,3} = 180$ V, Equation (2) yields

$$\frac{\xi_{0,3}}{f_3} = \frac{\xi_{0,1}}{f_1} \quad (4)$$

Solving Equation (4) for $f_3$, we find that

$$f_3 = \left(\frac{\xi_{0,3}}{\xi_{0,1}}\right)f_1 = \left(\frac{180 \text{ V}}{75 \text{ V}}\right)(280 \text{ Hz}) = 670 \text{ Hz}$$
2) The primary coil of a transformer is connected to a 120V/60Hz wall outlet. The secondary coil is connected to a lamp that dissipates 60 W of power. What is the rms current in the primary coil?

The secondary of the transformer delivers a power of 60 W. In the development of the transformer theory, we assumed the transformer was 100 percent efficient so the power at the primary must also be 60 W. Knowing that the power is related to the rms voltage and current by \( P = V_{rms}I_{rms} \) and that the rms voltage at the primary is 120 V, we can determine the rms current at the primary.

Solve:

\[ I_{rms} = \frac{P}{V_{rms}} = \frac{60 \text{ W}}{120 \text{ V}} = 0.50 \text{ A} \]

3) You are working with a generator that has an rms voltage of 15 V and you want the rms current through an inductor to be 0.610 A. What inductive reactance do you need your inductor to supply?

**REASONING** The rms voltage \( V_{rms} \) across the inductor is given by \( V_{rms} = I_{rms}X_L \) (Equation 23.3), where \( I_{rms} \) is the rms current in the circuit, and \( X_L \) is the inductive reactance. The inductor is the only circuit element connected to the generator, so the rms voltage across the inductor is equal to the rms generator voltage: \( V_{rms} = 15.0 \text{ V} \).

**SOLUTION** Solving Equation 23.3 for \( X_L \), we obtain

\[ X_L = \frac{V_{rms}}{I_{rms}} = \frac{15.0 \text{ V}}{0.610 \text{ A}} = 24.6 \Omega \]

4) The resonant frequency of an RCL circuit is 1.3 kHz when the value of the inductance is 7.0 mH. What is the resonant frequency when the inductance is 1.5 mH?

**REASONING** The resonant frequency of an RCL circuit is given by \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (Equation 23.10), where \( L \) is inductance and \( C \) is the capacitance. Because only the inductance of this circuit changes, from \( L_1 = 7.0 \text{ mH} \) to \( L_2 = 1.5 \text{ mH} \), we obtain the initial and final resonant frequencies from Equation 23.10:

\[ f_{01} = \frac{1}{2\pi \sqrt{L_1C}} \quad \text{and} \quad f_{02} = \frac{1}{2\pi \sqrt{L_2C}} \quad (1) \]

We will solve the first of Equations (1) for the capacitance \( C \), and substitute the result into the second of Equations (1).

**SOLUTION** Squaring both sides of the first of Equations (1) and solving for \( C \), we obtain

\[ (f_{01})^2 = \frac{1}{(2\pi)^2 L_1 C} \quad \text{or} \quad C = \frac{1}{(2\pi f_{01})^2 L_1} \quad (2) \]

Substituting Equation (2) into the second of Equations (1) yields

\[ f_{02} = \frac{1}{2\pi \sqrt{L_2C}} = \frac{1}{2\pi \sqrt{L_2 \left[ \frac{1}{(2\pi f_{01})^2 L_1} \right]}} = \frac{2\pi f_{01}}{2\pi \sqrt{L_2 \left[ \frac{L_2}{L_1} \right]}} = f_{01} \sqrt{\frac{L_1}{L_2}} \quad (3) \]

\[ f_{02} = f_{01} \sqrt{\frac{L_1}{L_2}} = (1.3 \text{ kHz}) \sqrt{\frac{7.0 \text{ mH}}{1.5 \text{ mH}}} = 2.8 \text{ kHz} \]