Chapter 31 Homework Solutions

1. **REASONING** For an element whose chemical symbol is X, the symbol for the nucleus is \( ^A_Z X \), where \( A \) represents the total number of protons and neutrons (the nucleon number) and \( Z \) represents the number of protons in the nucleus (the atomic number). The number of neutrons \( N \) is related to \( A \) and \( Z \) by Equation 31.1: \( A = Z + N \).

**SOLUTION** For the nucleus \( ^{208}_{82} \text{Pb} \), we have \( Z = 82 \) and \( A = 208 \).

a. The net electrical charge of the nucleus is equal to the total number of protons multiplied by the charge on a single proton. Since the \( ^{208}_{82} \text{Pb} \) nucleus contains 82 protons, the net electrical charge of the \( ^{208}_{82} \text{Pb} \) nucleus is

\[
q_{\text{net}} = (82)(+1.60 \times 10^{-19} \text{ C}) = +1.31 \times 10^{-17} \text{ C}
\]

b. The number of neutrons is \( N = A - Z = 208 - 82 = 126 \).

c. By inspection, the number of nucleons is \( A = 208 \).

d. The approximate radius of the nucleus can be found from Equation 31.2, namely

\[
r = (1.2 \times 10^{-15} \text{ m}) A^{1/3} = (1.2 \times 10^{-15} \text{ m})(208)^{1/3} = 7.1 \times 10^{-15} \text{ m}
\]

e. The nuclear density is the mass per unit volume of the nucleus. The total mass of the nucleus can be found by multiplying the mass \( m_{\text{nucleon}} \) of a single nucleon by the total number \( A \) of nucleons in the nucleus. Treating the nucleus as a sphere of radius \( r \), the nuclear density is

\[
\rho = \frac{m_{\text{total}}}{V} = \frac{m_{\text{nucleon}} A}{\frac{4}{3} \pi r^3} = \frac{m_{\text{nucleon}} A}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m}) A^{1/3}} = \frac{m_{\text{nucleon}}}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3}
\]

Therefore,

\[
\rho = \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg/m}^3
\]

3. **REASONING** For an element whose chemical symbol is X, the symbol for the nucleus is \( ^A_Z X \) where \( A \) represents the number of protons and neutrons (the nucleon number) and \( Z \) represents the number of protons (the atomic number) in the nucleus.

**SOLUTION**

a. The symbol \( ^{195}_{78} X \) indicates that the nucleus in question contains \( Z = 78 \) protons, and \( N = A - Z = 195 - 78 = 117 \) neutrons. From the periodic table, we see that \( Z = 78 \) corresponds to platinum, Pt.

b. Similar reasoning indicates that the nucleus in question is sulfur, S, and the nucleus contains \( N = A - Z = 32 - 16 = 16 \) neutrons.
c. Similar reasoning indicates that the nucleus in question is copper, \( Cu \), and the nucleus contains \( N = A - Z = 63 - 29 = 34 \) neutrons.

d. Similar reasoning indicates that the nucleus in question is boron, \( B \), and the nucleus contains \( N = A - Z = 11 - 5 = 6 \) neutrons.

e. Similar reasoning indicates that the nucleus in question is plutonium, \( Pu \), and the nucleus contains \( N = A - Z = 239 - 94 = 145 \) neutrons.

11. **SSM REASONING** To obtain the binding energy, we will calculate the mass defect and then use the fact that 1 u is equivalent to 931.5 MeV. The atomic mass given for \(^7\text{Li}\) includes the 3 electrons in the neutral atom. Therefore, when computing the mass defect, we must account for these electrons. We do so by using the atomic mass of 1.007825 u for the hydrogen atom \(^1\text{H}\), which also includes the single electron, instead of the atomic mass of a proton.

**SOLUTION** Noting that the number of neutrons is \( 7 - 3 = 4 \), we obtain the mass defect \( \Delta m \) as follows:

\[
\Delta m = \left(3 \times 1.007825 \text{ u}\right) + \left(4 \times 1.008665 \text{ u}\right) - 7.016003 \text{ u} = 4.2132 \times 10^{-2} \text{ u}
\]

Since 1 u is equivalent to 931.5 MeV, the binding energy is

\[
\text{Binding energy} = \left(4.2132 \times 10^{-2} \text{ u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 39.25 \text{ MeV}
\]

13. **REASONING AND SOLUTION** For \(^{202}\text{Hg}\) the mass of the separated nucleons is

\[
m = 80(1.007825 \text{ u}) + 122(1.008665 \text{ u}) = 203.683 \text{ u}
\]

The mass defect is then \( \Delta m = 203.683 \text{ u} - 201.970617 \text{ u} = 1.712513 \text{ u} \).

This corresponds to a total binding energy of \( \left(1.712513 \text{ u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 1595 \text{ MeV} \) and a binding energy per nucleon of \( \frac{1595 \text{ MeV}}{202 \text{ nucleons}} = 7.90 \text{ MeV/nucleon} \).

21. **SSM REASONING AND SOLUTION** The general form for \( \beta^- \) decay is

\[
\begin{array}{ccc}
\text{Parent nucleus} & \to & \text{Daughter nucleus} \\
A^Z_{\text{P}} & \to & A^{Z+1}_{\text{D}} + ^0_{-1}\text{e} \\
\beta^- \text{ particle (electron)}
\end{array}
\]

Therefore, the \( \beta^- \) decay process for \(^{35}\text{S}\) is \(^{35}_{16}\text{S} \to ^{35}_{17}\text{Cl} + ^0_{-1}\text{e} \).
23. **REASONING AND SOLUTION**  The mass of the products is

\[ m = 222.01757 \text{ u} + 4.00260 \text{ u} = 226.02017 \text{ u} \]

The mass defect for the decay is

\[ \Delta m = 226.02540 \text{ u} - 226.02017 \text{ u} = 0.00523 \text{ u} \]

which corresponds to an energy of

\[ (0.00523 \text{ u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 4.87 \text{ MeV} \]

33. **SSM REASONING AND SOLUTION**  The number of radioactive nuclei that remains in a sample after a time \( t \) is given by Equation 31.5, \( N = N_0 e^{-\lambda t} \), where \( \lambda \) is the decay constant. From Equation 31.6, we know that the decay constant is related to the half-life by \( \lambda = 0.693 / T_{1/2} \); therefore, \( \lambda = 0.693 / T_{1/2} \) and we can write

\[ \frac{N}{N_0} = e^{-0.693/T_{1/2} t} \quad \text{or} \quad \frac{t}{T_{1/2}} = -\frac{1}{0.693} \ln \left( \frac{N}{N_0} \right) \]

When the number of radioactive nuclei decreases to one-millionth of the initial number, \( N / N_0 = 1.00 \times 10^{-6} \); therefore, the number of half-lives is

\[ \frac{t}{T_{1/2}} = -\frac{1}{0.693} \ln (1.00 \times 10^{-6}) = 19.9 \]

35. **REASONING AND SOLUTION**  The amount remaining is 0.0100% = 0.000 100. We know \( N / N_0 = e^{-0.693 t / T_{1/2}} \). Therefore, we find

\[ t = -\frac{T_{1/2}}{0.693} \ln \left( \frac{N}{N_0} \right) = -\frac{29.1 \text{ yr}}{0.693} \ln(0.000 100) = 387 \text{ yr} \]

37. **SSM REASONING AND SOLUTION**  According to Equation 31.5, \( N = N_0 e^{-\lambda t} \), the decay constant is

\[ \lambda = -\frac{1}{t} \ln \left( \frac{N}{N_0} \right) = -\frac{1}{20 \text{ days}} \ln \left( \frac{8.14 \times 10^{14}}{4.60 \times 10^{15}} \right) = 0.0866 \text{ days}^{-1} \]

The half-life is, from Equation 31.6,

\[ T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.0866 \text{ days}^{-1}} = 8.00 \text{ days} \]

47. **SSM REASONING AND SOLUTION**  The answer can be obtained directly from Equation 31.5, combined with Equation 31.6:

\[ \frac{N}{N_0} = e^{-\lambda t} = e^{-0.693 t / T_{1/2}} = e^{-0.693(41000 \text{ yr})/(5730 \text{ yr})} = 0.0070 \]

The percent of atoms remaining is \( 0.70 \% \).