Physics 102: Chapter 18 Homework Solutions

1. **REASONING AND SOLUTION** The total charge of the electrons is

\[ q = N(-e) = (6.0 \times 10^{13})(-1.60 \times 10^{-19} \text{ C}) \]

\[ q = -9.6 \times 10^{-6} \text{ C} = -9.6 \mu\text{C} \]

The net charge on the sphere is, therefore,

\[ q_{\text{net}} = +8.0 \mu\text{C} - 9.6 \mu\text{C} = -1.6 \mu\text{C} \]

3. **REASONING**
   a. Since the objects are metallic and identical, the charges on each combine and produce a net charge that is shared equally by each object. Thus, each object ends up with one-fourth of the net charge.

   b. The number of electrons (or protons) that make up the final charge on each object is equal to the final charge divided by the charge of an electron (or proton).

**SOLUTION**
   a. The net charge is the algebraic sum of the individual charges. The charge \( q \) on each object after contact and separation is one-fourth the net charge, or

   \[ q = \frac{1}{4}(1.6 \mu\text{C} + 6.2 \mu\text{C} - 4.8 \mu\text{C} - 9.4 \mu\text{C}) = -1.6 \mu\text{C} \]

   b. Since the charge on each object is negative, the charge is comprised of electrons. The number of electrons on each object is the charge \( q \) divided by the charge \(-e\) of a single electron:

   \[ \text{Number of electrons} = \frac{q}{-e} = \frac{-1.6 \times 10^{-6} \text{ C}}{-1.60 \times 10^{-19} \text{ C}} = 1.0 \times 10^{13} \]

6. **REASONING** The conservation of electric charge states that, during any process, the net electric charge of an isolated system remains constant (is conserved). Therefore, the net charge \( q_1 + q_2 \) on the two spheres before they touch is the same as the net charge after they touch. When the two identical metal spheres touch, the net charge will spread out equally over both of them. When the spheres are separated, the charge on each is the same.

**SOLUTION**
   a. Since the final charge on each sphere is \(+5.0 \mu\text{C}\), the final net charge on both spheres is \(2(+5.0 \mu\text{C}) = +10.0 \mu\text{C}\). The initial net charge must also be \(+10.0 \mu\text{C}\). The only spheres whose net charge is \(+10.0 \mu\text{C}\) are \(\text{B \ (q_B = -2.0 \mu\text{C}) and \ D \ (q_D = +12.0 \mu\text{C})}\)

   b. Since the final charge on each sphere is \(+3.0 \mu\text{C}\), the final net charge on the three spheres is \(3(+3.0 \mu\text{C}) = +9.0 \mu\text{C}\). The initial net charge must also be \(+9.0 \mu\text{C}\). The only spheres whose net charge is \(+9.0 \mu\text{C}\) are
c. Since the final charge on a given sphere in part (b) is +3.0 \( \mu C \), we would have to add \(-3.0 \mu C\) to make it electrically neutral. Since the charge on an electron is \(-1.6 \times 10^{-19} \text{ C}\), the number of electrons that would have to be added is

\[
\text{Number of electrons} = \frac{-3.0 \times 10^{-6} \text{ C}}{-1.6 \times 10^{-19} \text{ C}} = 1.9 \times 10^{13}
\]

8. **REASONING** The magnitude \( F \) of the forces that point charges \( q_1 \) and \( q_2 \) exert on each other varies with the distance \( r \) separating them according to \( F = \frac{k |q_1||q_2|}{r^2} \) (Equation 18.1), where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \).

We note that both charges are given in units of microcoulombs (\( \mu \text{C} \)), rather than the base SI units of coulombs (\( \text{C} \)). We will replace the prefix \( \mu \) with \( 10^{-6} \) when calculating the distance \( r \) from Equation 18.1.

**SOLUTION** Solving \( F = \frac{k |q_1||q_2|}{r^2} \) (Equation 18.1) for the distance \( r \), we obtain

\[
r^2 = k \frac{|q_1||q_2|}{F} \quad \text{or} \quad r = \sqrt{k \frac{|q_1||q_2|}{F}}
\]

Therefore, when the force magnitude \( F \) is 0.66 N, the distance between the charges must be

\[
r = \sqrt{\frac{k |q_1||q_2|}{F}} = \sqrt{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(8.4 \times 10^{-6} \text{ C}\right)\left(5.6 \times 10^{-6} \text{ C}\right)} = 0.80 \text{ m}
\]

11. **SSM WWW REASONING** Initially, the two spheres are neutral. Since negative charge is removed from the sphere which loses electrons, it then carries a net positive charge. Furthermore, the neutral sphere to which the electrons are added is then negatively charged. Once the charge is transferred, there exists an electrostatic force on each of the two spheres, the magnitude of which is given by Coulomb's law (Equation 18.1), \( F = \frac{k |q_1||q_2|}{r^2} \).

**SOLUTION**

a. Since each electron carries a charge of \(-1.60 \times 10^{-19} \text{ C}\), the amount of negative charge removed from the first sphere is

\[
(3.0 \times 10^{13} \text{ electrons}) \left(\frac{1.60 \times 10^{-19} \text{ C}}{1 \text{ electron}}\right) = 4.8 \times 10^{-6} \text{ C}
\]

Thus, the first sphere carries a charge \(+4.8 \times 10^{-6} \text{ C}\), while the second sphere carries a charge \(-4.8 \times 10^{-6} \text{ C}\). The magnitude of the electrostatic force that acts on each sphere is, therefore,

\[
F = \frac{k |q_1||q_2|}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(4.8 \times 10^{-6} \text{ C}\right)^2}{(0.50 \text{ m})^2} = 0.83 \text{ N}
\]
b. Since the spheres carry charges of opposite sign, the force is **attractive**.

14. **REASONING** The electrical force that each charge exerts on charge 2 is shown in the following drawings. \( \mathbf{F}_{21} \) is the force exerted on 2 by 1, and \( \mathbf{F}_{23} \) is the force exerted on 2 by 3. Each force has the same magnitude, because the charges have the same magnitude and the distances are equal.

The net electric force \( \mathbf{F} \) that acts on charge 2 is shown in the following diagrams.

It can be seen from the diagrams that the largest electric force occurs in (a), followed by (c), and then by (b).

**SOLUTION** The magnitude \( F_{21} \) of the force exerted on 2 by 1 is the same as the magnitude \( F_{23} \) of the force exerted on 2 by 3, since the magnitudes of the charges are the same and the distances are the same. Coulomb’s law gives the magnitudes as

\[
F_{21} = F_{23} = \frac{k|q_1||q_2|}{r^2}
\]

\[
= \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{} \right) \left( 8.6 \times 10^{-6} \text{ C} \right) \left( 8.6 \times 10^{-6} \text{ C} \right) \left( 3.8 \times 10^{-3} \text{ m} \right)^2
= 4.6 \times 10^4 \text{ N}
\]

In part (a) of the drawing showing the net electric force acting on charge 2, both \( \mathbf{F}_{21} \) and \( \mathbf{F}_{23} \) point to the left, so the net force has a magnitude of

\[
F = 2F_{12} = 2 \left( 4.6 \times 10^4 \text{ N} \right) = 9.2 \times 10^4 \text{ N}
\]

In part (b) of the drawing showing the net electric force acting on charge 2, \( \mathbf{F}_{21} \) and \( \mathbf{F}_{23} \) point in opposite directions, so the net force has a magnitude of \( 0 \text{ N} \).
In part (c) showing the net electric force acting on charge 2, the magnitude of the net force can be obtained from the Pythagorean theorem:

\[ F = \sqrt{F_{21}^2 + F_{23}^2} = \sqrt{\left(4.6 \times 10^4 \text{ N}\right)^2 + \left(4.6 \times 10^4 \text{ N}\right)^2} = 6.5 \times 10^4 \text{ N} \]

16. **REASONING AND SOLUTION** The electrostatic forces decreases with the square of the distance separating the charges. If this distance is increased by a factor of 5 then the force will decrease by a factor of 25. The new force is, then,

\[ F = \frac{3.5 \text{ N}}{25} = 0.14 \text{ N} \]

29. **SOLUTION** Knowing the electric field at a spot allows us to calculate the force that acts on a charge placed at that spot, without knowing the nature of the object producing the field. This is possible because the electric field is defined as \( E = \frac{F}{q_0} \), according to Equation 18.2. This equation can be solved directly for the force \( F \), if the field \( E \) and charge \( q_0 \) are known.

**SOLUTION** Using Equation 18.2, we find that the force has a magnitude of

\[ F = E |q_0| = \left(260 \,000 \,\text{N/C}\right) \left(7.0 \times 10^{-6} \,\text{C}\right) = 1.8 \,\text{N} \]

If the charge were positive, the direction of the force would be due west, the same as the direction of the field. But the charge is negative, so the force points in the opposite direction or due east. Thus, the force on the charge is \( 1.8 \,\text{N due east} \).

35. **REASONING AND SOLUTION**

a. In order for the field to be zero, the point cannot be between the two charges. Instead, it must be located on the line between the two charges on the side of the positive charge and away from the negative charge. If \( x \) is the distance from the positive charge to the point in question, then the negative charge is at a distance \( (3.0 \,\text{m} + x) \) meters from this point. For the field to be zero here we have

\[ \frac{k|q_-|}{(3.0 \,\text{m} + x)^2} = \frac{k|q_+|}{x^2} \quad \text{or} \quad \frac{|q_-|}{(3.0 \,\text{m} + x)^2} = \frac{|q_+|}{x^2} \]

Solving for the ratio of the charge magnitudes gives

\[ \frac{|q_-|}{|q_+|} = \frac{16.0 \,\mu\text{C}}{4.0 \,\mu\text{C}} = \frac{(3.0 \,\text{m} + x)^2}{x^2} \quad \text{or} \quad 4.0 = \frac{(3.0 \,\text{m} + x)^2}{x^2} \]

Suppressing the units for convenience and rearranging this result gives

\[ 4.0x^2 = (3.0 + x)^2 \quad \text{or} \quad 4.0x^2 = 9.0 + 6.0x + x^2 \quad \text{or} \quad 3x^2 - 6.0x - 9.0 = 0 \]

Solving this quadratic equation for \( x \) with the aid of the quadratic formula (see Appendix C.4) shows that

\( x = 3.0 \,\text{m} \quad \text{or} \quad x = -1.0 \,\text{m} \)
We choose the positive value for \( x \), since the negative value would locate the zero-field spot between the two charges, where it can not be (see above). Thus, we have \( x = 3.0 \text{ m} \).

b. Since the field is zero at this point, the force acting on a charge at that point is 0 N.

### 36. REASONING

a. The magnitude \( E \) of the electric field is given by \( E = \sigma / \varepsilon_0 \) (Equation 18.4), where \( \sigma \) is the charge density (or charge per unit area) and \( \varepsilon_0 \) is the permittivity of free space.

b. The magnitude \( F \) of the electric force that would be exerted on a \( K^+ \) ion placed inside the membrane is the product of the magnitude \( |q_0| \) of the charge and the magnitude \( E \) of the electric field (see Equation 18.2), or \( F = |q_0|E \).

### SOLUTION

a. The magnitude of the electric field is

\[
E = \frac{\sigma}{\varepsilon_0} = \frac{7.1 \times 10^{-6} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 8.0 \times 10^5 \text{ N/C}
\]

b. The magnitude \( F \) of the force exerted on a \( K^+ \) ion \((q_0 = +e)\) is

\[
F = |q_0|E = |e|E = |1.60 \times 10^{-19} \text{ C}|(8.0 \times 10^5 \text{ N/C}) = 1.3 \times 10^{-13} \text{ N}
\]